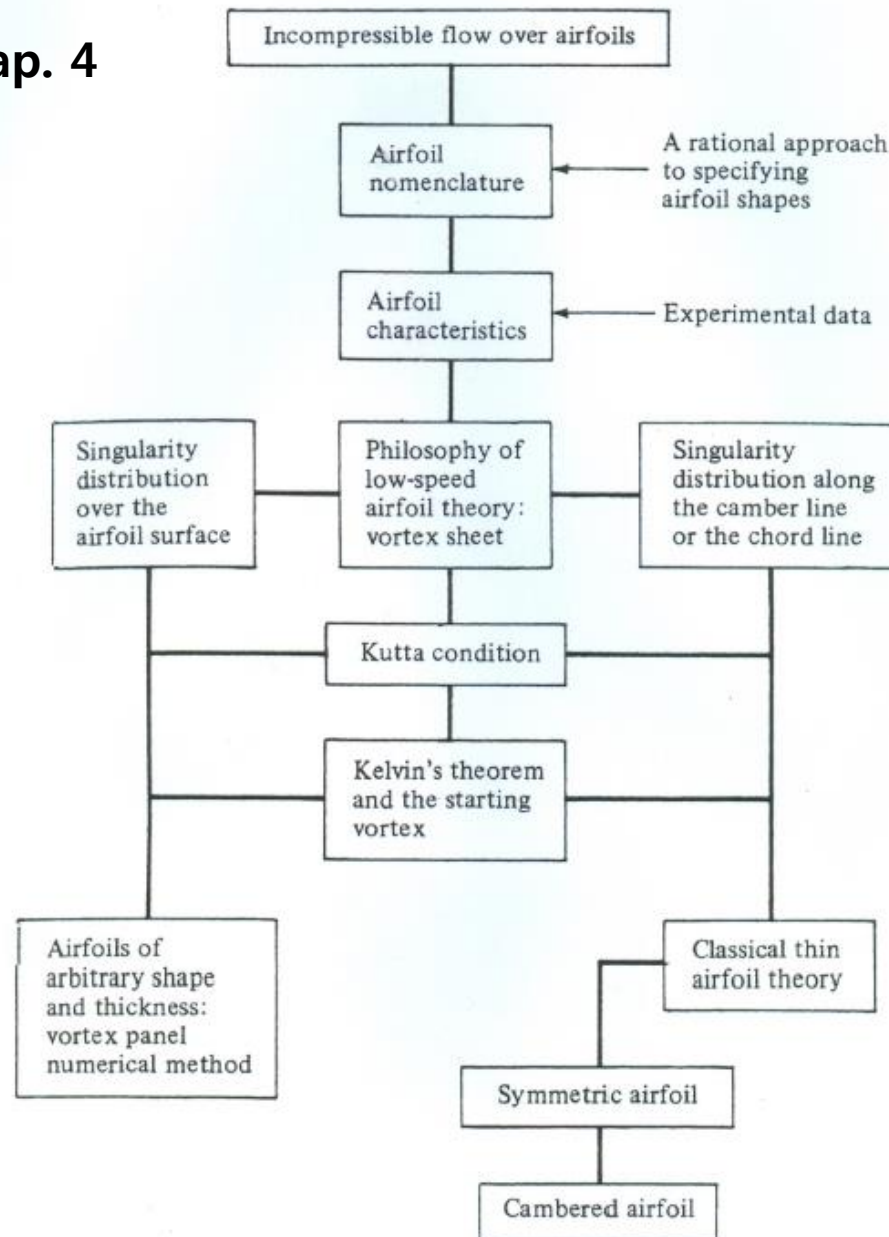


Incompressible Flow over Airfoils

Road map for Chap. 4

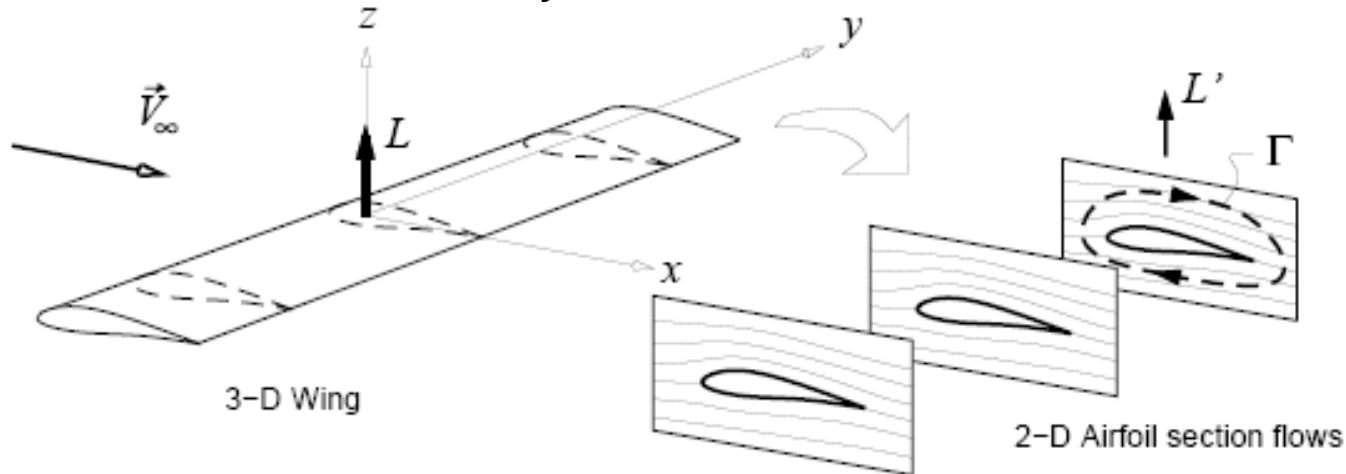


Incompressible Flow over Airfoils

< 4.1 Introduction >

❖ Incompressible flow over airfoils

- Prandtl (20C 초) → Airfoil (2D)
- Wing (3D)
- Body

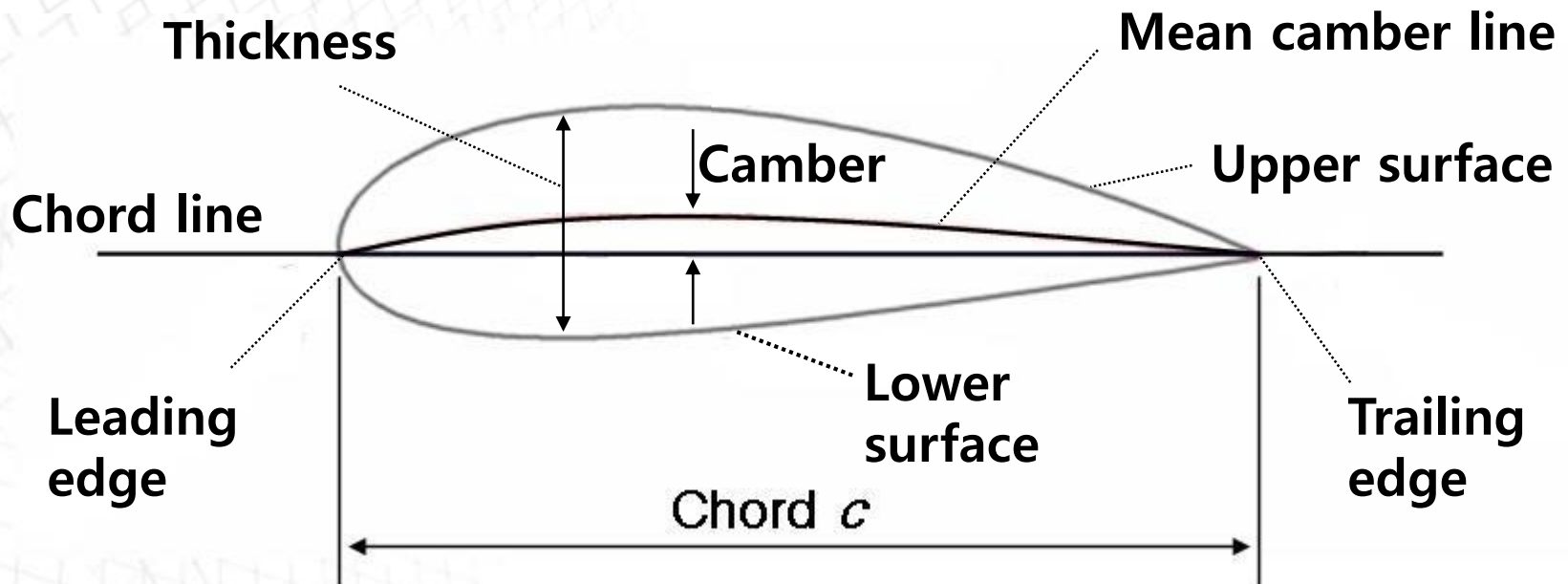


Airfoil : any section of the wing cut by a plane normal to y -axis

Incompressible Flow over Airfoils

< 4.2 Airfoil Nomenclature >

❖ **NACA** (National Advisory Committee for Aeronautics) **series**



< 4.2 Airfoil Nomenclature >

❖ **NACA (National Advisory Committee for Aeronautics) series**

NACA 4-digit series

* NACA2412

2 : max. camber = 2% of the chord

4 : the location of max. camber = 40% of the chord

12 : max. thickness = 12% of the chord

If the airfoil is symmetric, it becomes NACA00XX

NACA 5-digit series

* NACA23012

2 : $2 \cdot 0.3 / 2 = 0.3$ design C_L

30 : $30 / 2 \% =$ the location of max. camber

12 : max. thickness = 12% of the chord

< 4.2 Airfoil Nomenclature >

❖ **NACA** (National Advisory Committee for Aeronautics) **Series**

6-digit series laminar flow airfoil

* NACA65-218

6 : series designation

5 : min. pressure location = 50% of the chord

2 : design $C_L = 0.2$

18 : max. thickness = 18% of the chord

Other notations

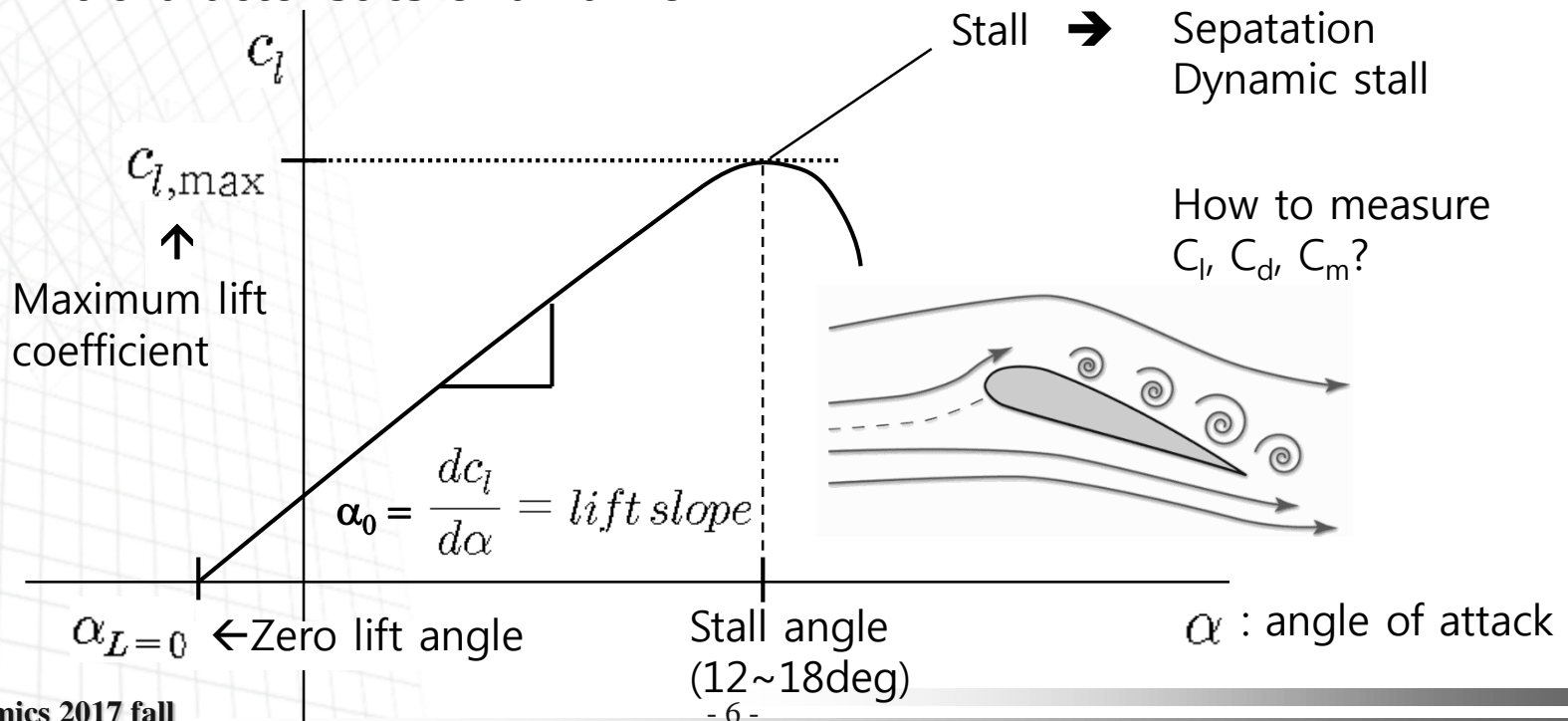
* SC1095

* VR12

Incompressible Flow over Airfoils

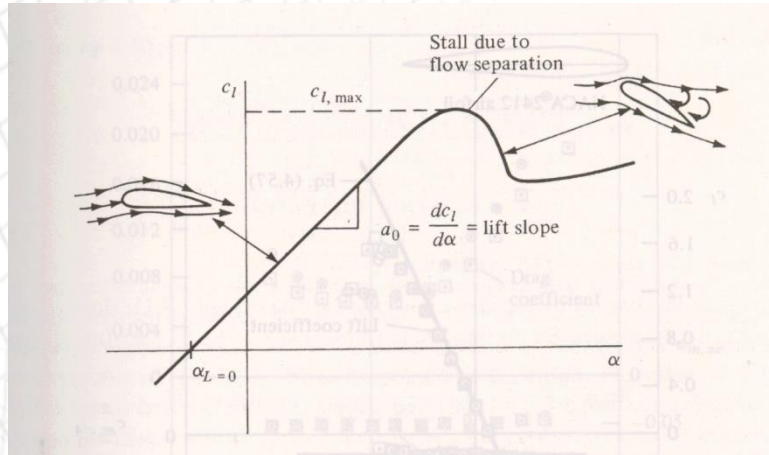
< 4.3 Airfoil Characteristics >

- * 1930~40 NASA carried numerous experiments on NACA airfoil characteristics (Measured C_l , C_d , $C_m \rightarrow$ 2-D data)
- * In the future, new airfoils should be designed and tested (consideration of aerodynamic, dynamic & acoustic limitation)
- * Typical lift characteristics of an airfoil



Incompressible Flow over Airfoils

< 4.3 Airfoil Characteristics >



[Def.]

α , angle of attack : the angle between the freestream velocity and the chord

[Note]

1. α_0 is not usually a function of Re.
2. $C_{l, \max}$ is dependent on Re.

< 4.3 Airfoil Characteristics >

❖ Typical drag & pitching moment characteristics

* Aerodynamic drag = Pressure drag + Skin friction drag
 ↓
 Sensitive to Re.
 (form drag) Profile drag

* AC (Aerodynamic Center)

[Def.] The point about which the moment is independent of AOA

→ Subsonic : $AC=c/4$
→ Supersonic : $AC=c/2$

Incompressible Flow over Airfoils

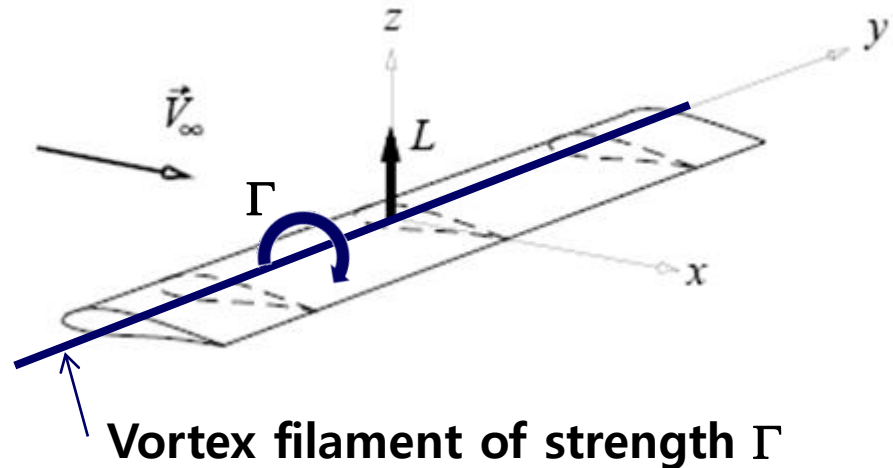
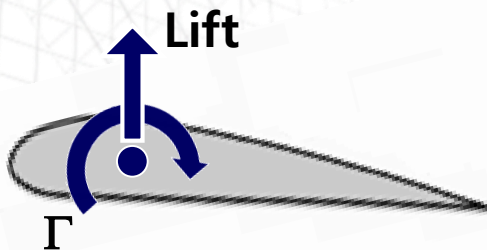
< 4.4 Vortex Sheet >

❖ Kutta-Joukowski Theorem

- * Kutta (German), Joukowski(Russia)
- * Incompressible, inviscid flow

$$L = \rho_{\infty} v_{\infty} \Gamma$$

- * Γ : positive clockwise



Incompressible Flow over Airfoils

< 4.4 Vortex Sheet >

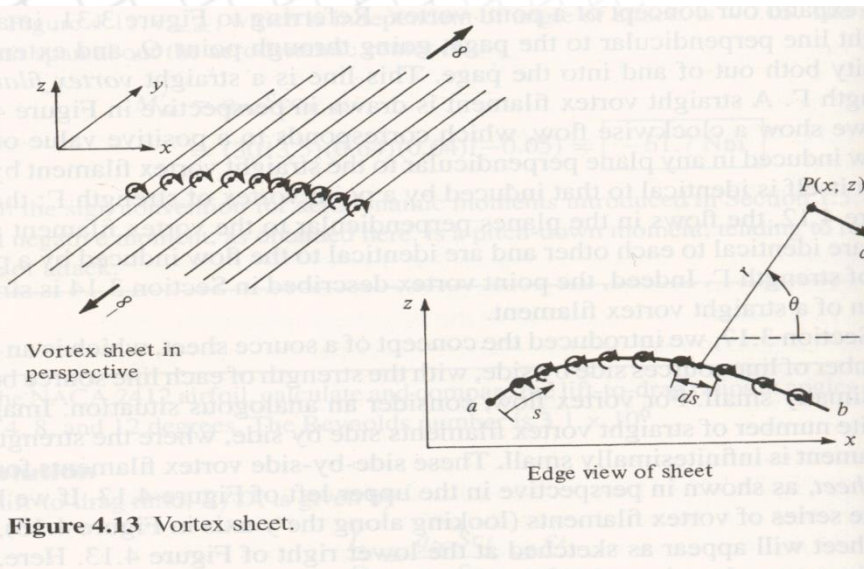


Figure 4.13 Vortex sheet.

* $\gamma(s)$ = the strength of vortex sheet per unit length along s

* From Biot-Savart Law

$$dV_{\theta} = -\frac{\gamma ds}{2\pi r}$$

* Velocity potential for vortex flow

$$V_{\theta} = \frac{1}{r} \frac{\partial \phi}{\partial \theta} \rightarrow d\phi = -\frac{\gamma ds}{2\pi} \theta$$

* Velocity potential at P

$$\rightarrow \phi(x, z) = -\frac{1}{2\pi} \int_a^b \theta \gamma ds$$

Incompressible Flow over Airfoils

< 4.4 Vortex Sheet >

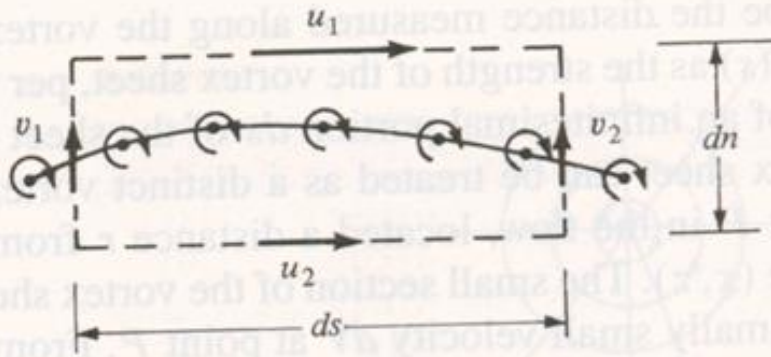


Figure 4.14 Tangential velocity jump across a vortex sheet.

* Circulation around the dashed path

$$\begin{aligned}\Gamma_1 &= \gamma ds = \oint \vec{V} d\vec{l} \\ &= - \{ (v_2 - v_1) dn + (u_2 - u_1) ds \} \\ &= (v_1 - v_2) dn + (u_1 - u_2) ds\end{aligned}$$

* If $dn \rightarrow 0$ $\gamma ds = (u_1 - u_2) ds \quad \rightarrow \quad \therefore \gamma = u_1 - u_2$

(Note)

The local strength of the vortex sheet is equal to the difference (jump) in tangential velocity across the vortex sheet

Incompressible Flow over Airfoils

< 4.4 Vortex Sheet >

* "Vortex Sheet" - Application for inviscid, incompressible flow

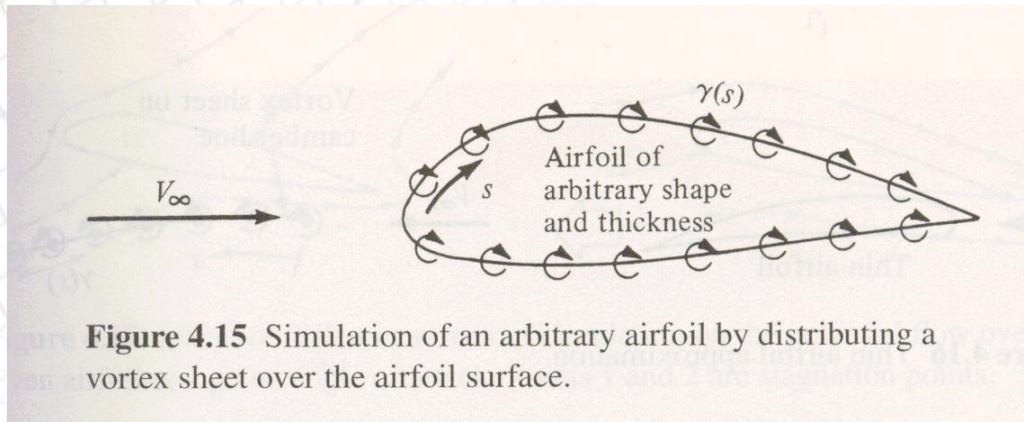


Figure 4.15 Simulation of an arbitrary airfoil by distributing a vortex sheet over the airfoil surface.

* Calculate $g(s)$ to form the streamlines with a give airfoil shape

(Note)

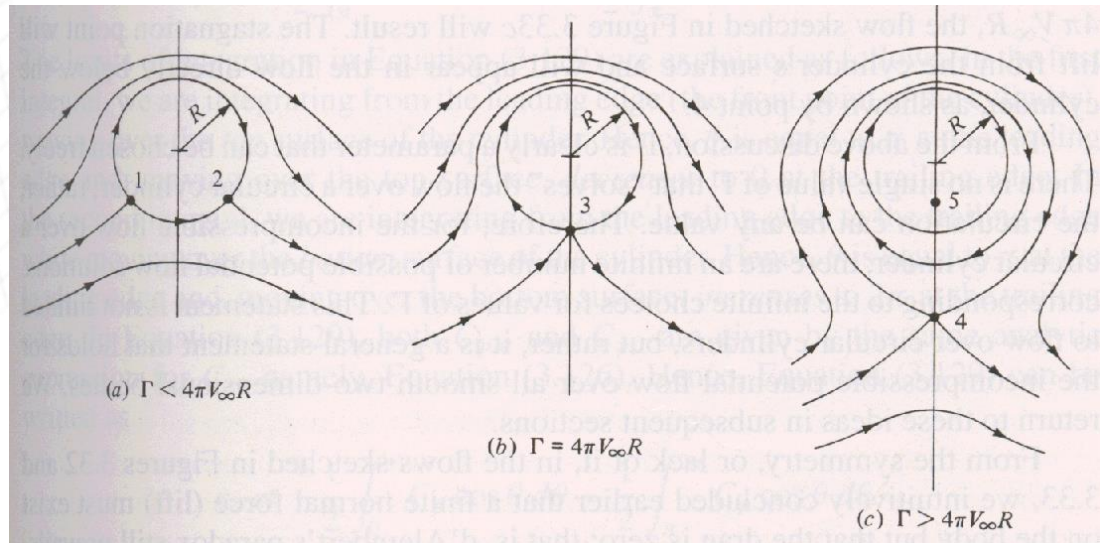
"Vortex sheet method" is more than just a mathematical device; it also has a physical meaning

ex. : Replacing the boundary layer ($\nabla \times \vec{V} \neq 0$) with a vortex sheet

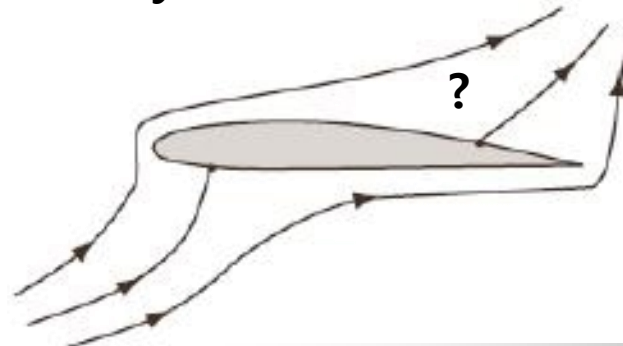
Incompressible Flow over Airfoils

< 4.5 The Kutta Condition >

* For a circular cylinder,



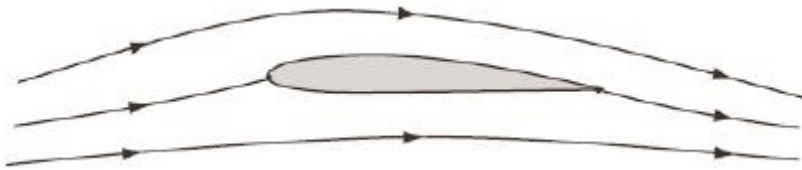
* For a given α , \rightarrow should have only one solution



Incompressible Flow over Airfoils

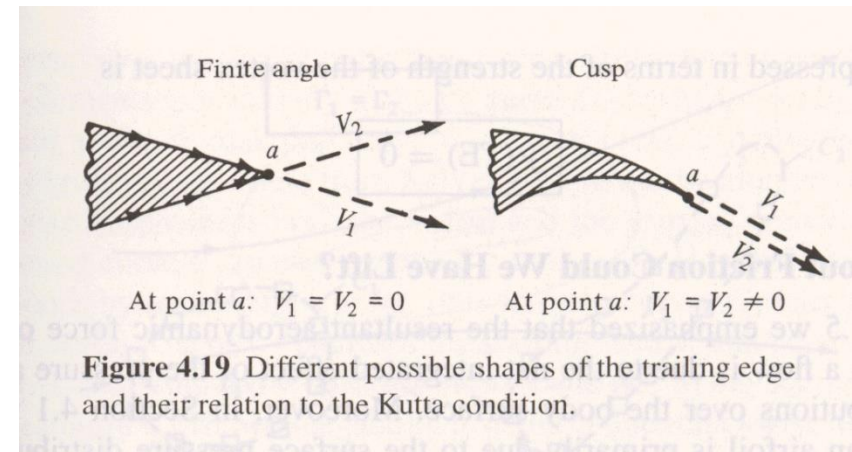
< 4.5 The Kutta Condition >

* From the experiments, we know that the velocity at the trailing-edge in finite. → Kutta Condition



$$\gamma(\text{TE}) = V_1 - V_2 = 0$$

$$V(\text{TE}) = \text{finite}$$



* The circulation around the airfoil is the value to ensure that the flow smoothly leaves the trailing edge.

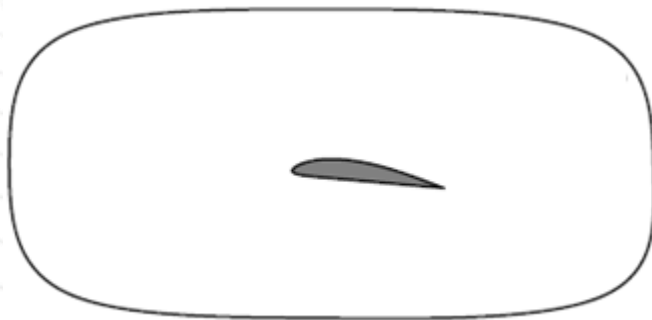
Incompressible Flow over Airfoils

< 4.6 Kelvin's Circulation Theorem >

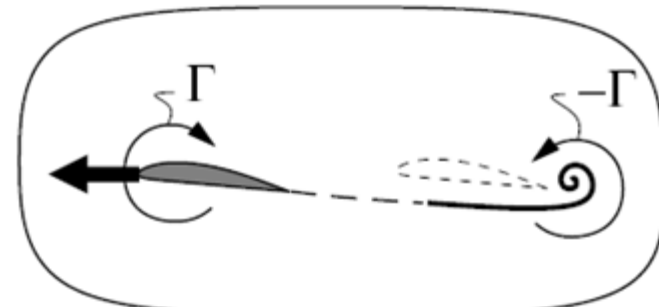
- * Assume)
1. Inviscid
 2. Incompressible
 3. No body forces

$\frac{D\Gamma}{Dt} = 0$ → The time rate of change of circulation around a closed curve consisting of the same fluid elements is zero

Ex) Starting vortex



[at rest]



[after the start]